Measurement of parity violation in $n + {}^3\text{He} \rightarrow p + t$: Geometry Factors and Optimizations using Monte Carlo

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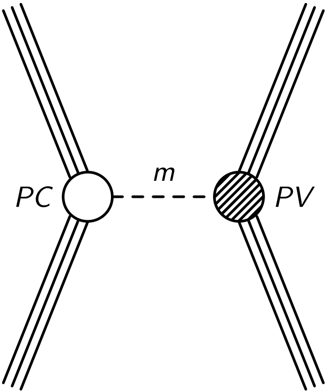
April 13, 2015

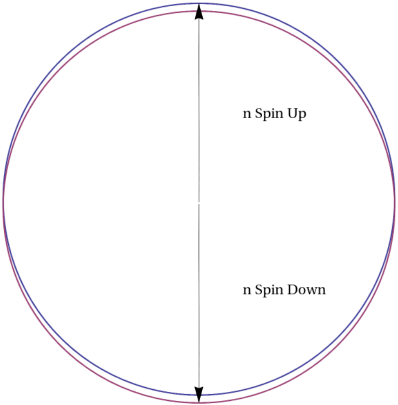
Experimental Details

Hadronic Weak Interaction



Here is an example of a hadronic weak interaction. In the vertex on the left, a nucleon strongly couples to a meson (in the model we use, this is a π, ρ, or ω). In the vertex on the right, the meson will convert to a weak boson, which will in turn couple weakly to the other nucleon.

Proton Asymmetry



If the polarization of the neutrons is precisely controlled, the parity of the reaction can be observed. Since weak interactions do not conserve parity, any weak coupling will produce an asymmetric distribution in the reaction products. So we can use the measurement of the asymmetry as a test of the strength of these weak couplings.

Angular Correlation

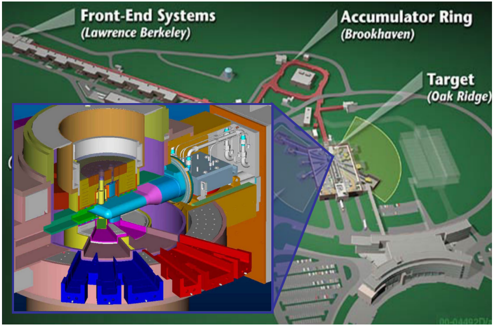
The experimentally measured asymmetry α_{phys} arises from the observable $\sigma_n \cdot k_p$, where σ_n is the direction of the neutron spin, and k_p is the direction of the proton's momentum. In this experiment, the spin orientation of the neutron is controlled through polarization and spin rotation. So the parity-violating asymmetry is a function of this product:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi}[1 + f(\sigma_n \cdot k_p)]$$

The resulting asymmetry in the distribution of proton momentum is how the asymmetry is measured. Since the neutron spins will be always be oriented along either $\pm\hat{z}$, we can simplify the expression for the asymmetry by using $\cos\theta$, where θ is the angle emission angle with respect to $\hat{z}$ and using $\pm$ to specify the spin state.

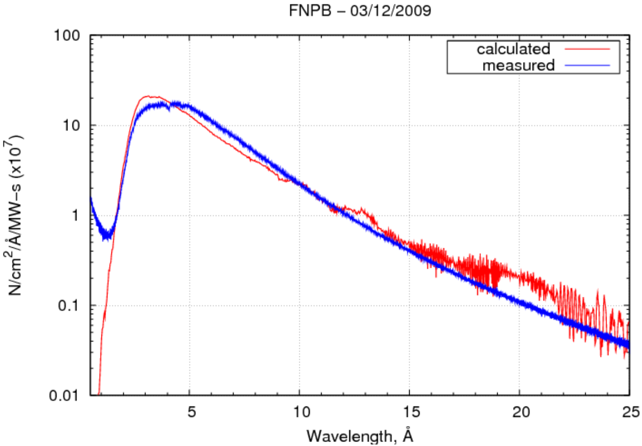
$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi}(1 \pm \alpha_{phys} \cos\theta)$$

Spallation Neutron Source



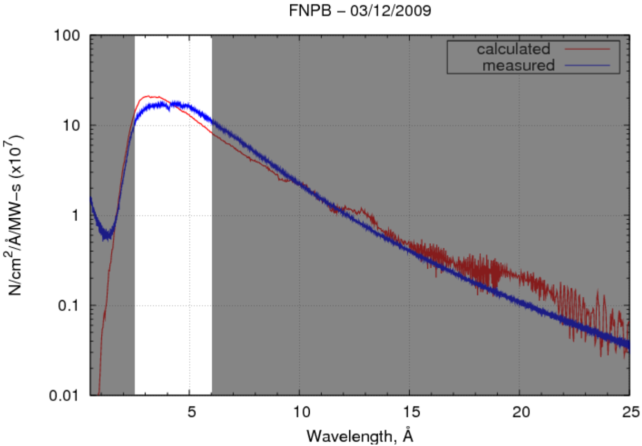
The Spallation Neutron source uses a series of proton accelerators and a proton ring to generate bunches of protons with an energy of 1 GeV. These protons are produced with a frequency of 60 Hz, and collided with a liquid Hg target. The resulting spallation produces neutrons which are sent into an array of guides for use in different experiments.

Fundamental Neutron Physics Beamline



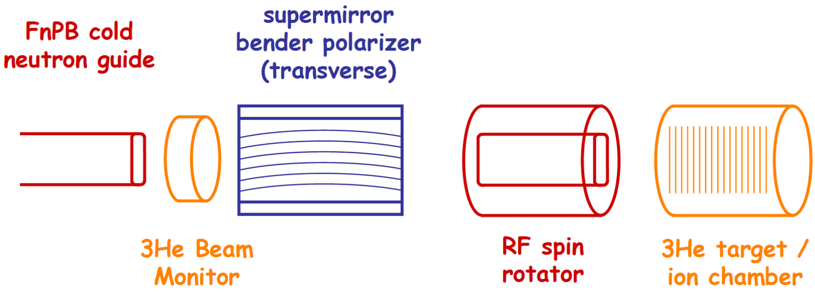
The neutron spectrum that travels down FNPB guide looks like this.

Neutron Spectrum



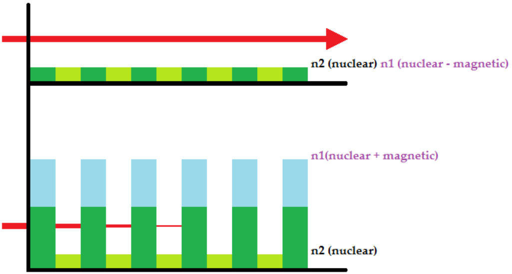
A pair of choppers block neutrons outside of the peak intensity range. The resulting spectrum has energies from approximately 2.5Å to 6Å.

Instrument Diagram



The experimental volume is contained inside large magnetic field coils which produce a very uniform field along the neutron flight path of 9.14G. The neutrons exiting the polarizer are aligned with this field and will maintain their spin state unless they are rotated.

Supermirror Polarizer



The neutrons are polarized by a combination of a nuclear and magnetic scattering. At the energy scale of the neutrons used in this experiment, they demonstrate optical behavior and can be manipulated with an "index of refraction." By layering materials with different nucler and magnetic cross sections, neutrons of one spin state can be reflected and absorbed, so the transmitted beam is polarized in the other state.

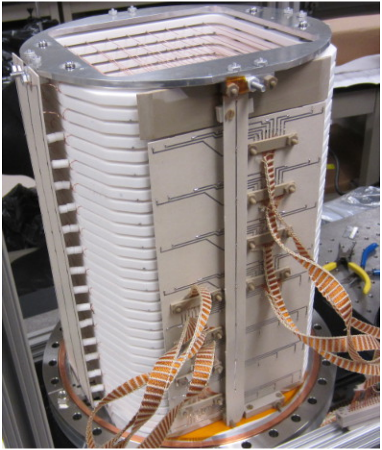
Neutron Spin Rotator

In order to measure the asymmetry, the distribution of reaction products must be compared for two opposite spin states. So we will use a radio-frequency spin rotator to "flip" the spin orientation of all the neutrons in every other pulse, and then compare the difference.

Neutrons in the holding field will undergo precession about the field axis due to the torque $\tau = \vec{\mu} \times \vec{B}$ on their magnetic field moment. This precession frequency, $\omega = \frac{2\mu B}{\hbar}$, determines the resonance condition necessay for an applied field to rotate the spin state of the neutron.

As neutrons pass through the spin rotator volume, their spin orientation will rotate according to the time spent in the volume. So the amplitude of the rotator field must be different for different neutron velocities. We can take advantage of the correlation between time of flight and energy of the neutron in order to achieve this. If the amplitude of the rotation field is adjusted proportionally to $\frac{1}{t}$, neutrons of different velocities will all undergo a rotation of π in the spin flipper volume.

Wire Chamber



The ion chamber contains both the target and detector – ^3He gas at 0.476 atmosphere. Inside the vessel is a wire stack which contains the signal and voltage wires which will be used to collect the ions.

Simulation

The Geometry Factor

The reaction products can be emitted in any direction in the ion chamber. So the dot product that characterizes the strength of the asymmetry, $\cos\theta$, can range from 0 (if the products are emitted in the $x-y$ plane) to 1 (if the products are emitted along $\hat{z}$). Depending on their location in the ion chamber, different cells will have a varying sensitivy to the physics asymmetry, which is a function of the location and time of the measurement and is a weighted average of all events that contribute energy in that element.

We will call that mean sensitivity the geometry factor and define it as:

$$G_\kappa \equiv \frac{\langle E^\kappa \cos\theta \rangle}{\langle E^\kappa \rangle}$$

Simulation Objectives

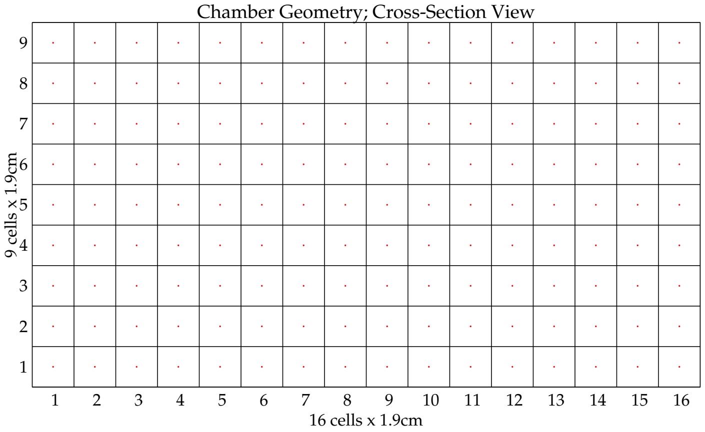
Some desired simulation objectives:

-Calculated geometry factors
-Optimized pressure
-Optimized collimation
-Estimated running time / uncertainty

In order to construct a successful simulation, one must find the best compromise between complex physics and fast calculations. It also should be scalable and able to take advantage of parallel resources. A custom code will allow the best approximations to be made where available for a given system.

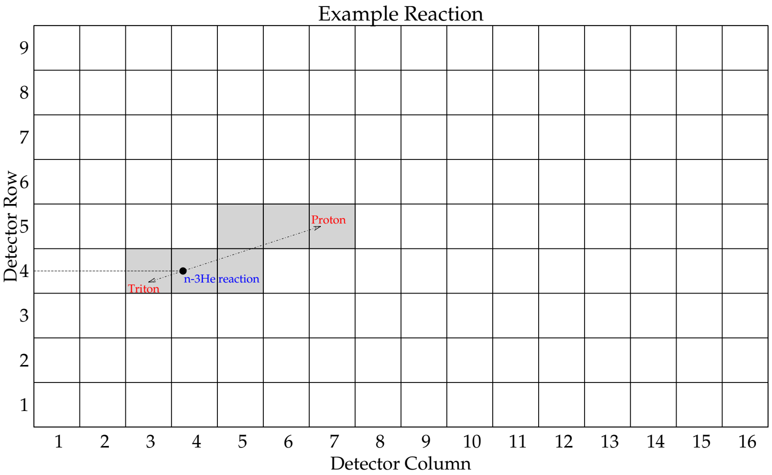
Cell Model in Simulation

Neutron beam is incident from left, in the $+\hat{z}$ direction.



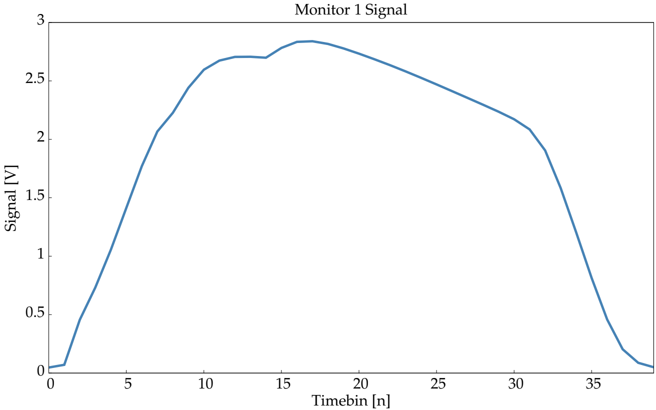
View in the yz-plane of the wire chamber.

Tracking Matrix



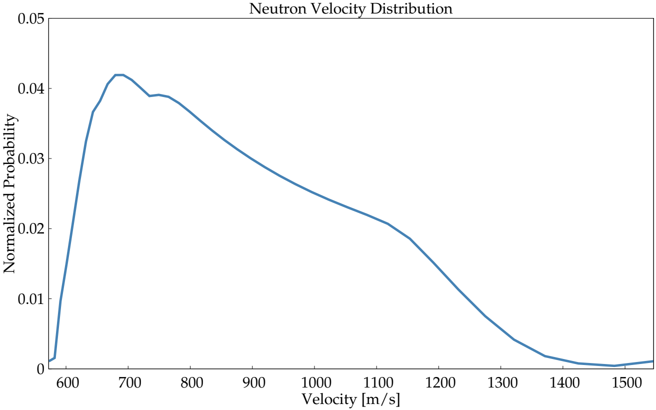
The proton carries 573 keV away from the reaction, and the triton carries 191 keV. These products will ionize the ^{3}He and travel a total of 12 cm in the gas.

Time Signal



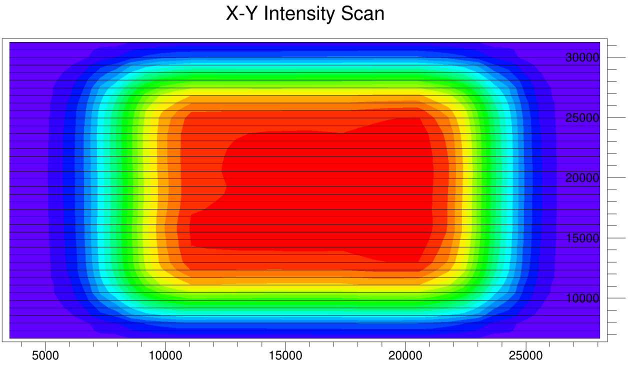
Shape of single 60Hz pulse as measured by the gas monitor.

Neutron Velocity Distribution



Velocity distribution generated from time-of-flight analysis:
$v = \frac{15.15}{.0098 + \frac{tbin}{2400}}$

Physical Beam Profile

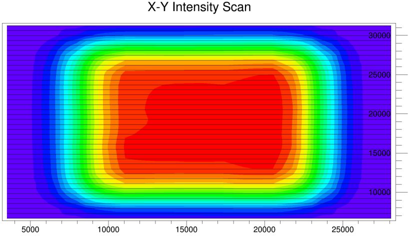


The beam is scanned on a grid to determine the centroid and shape.[1] Shown here is the upstream scan, right after the neutrons exit the guide aperture.

[1]Beam divergence can also be modeled.

Physical Beam Profile

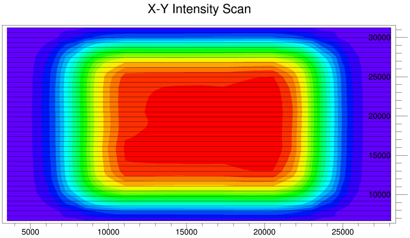
Let's try to see if we can simplify this model in order to produce a more efficient generator. If we decompose the intensity array, the first five sigma values are:

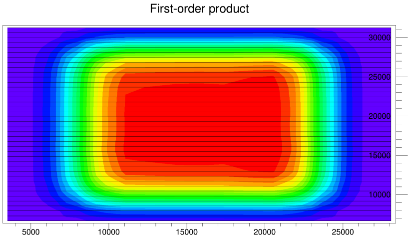


$$\Sigma_1 = 1.634$$
$$\Sigma_2 = 0.016$$
$$\Sigma_3 = 0.004$$
$$\Sigma_4 = 0.002$$
$$\Sigma_5 = 0.001$$

So we can include only the first-order vectors, and retain 98.5% of the information.

Physical Beam Profile

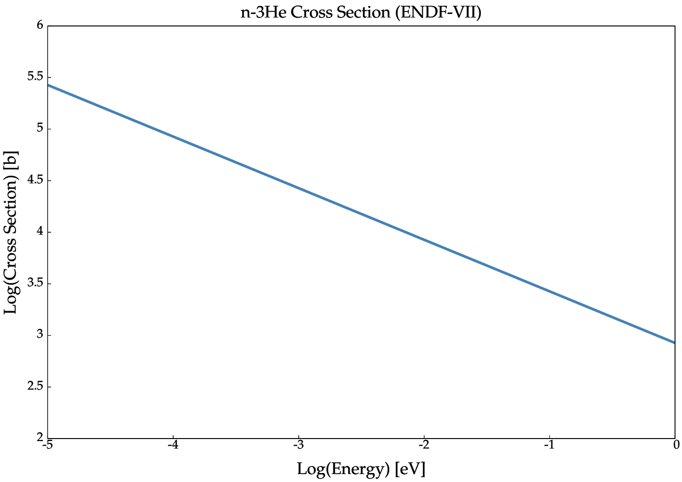




Now the image on the right shows the simulated beam profile. $\chi^2 = 0.01$

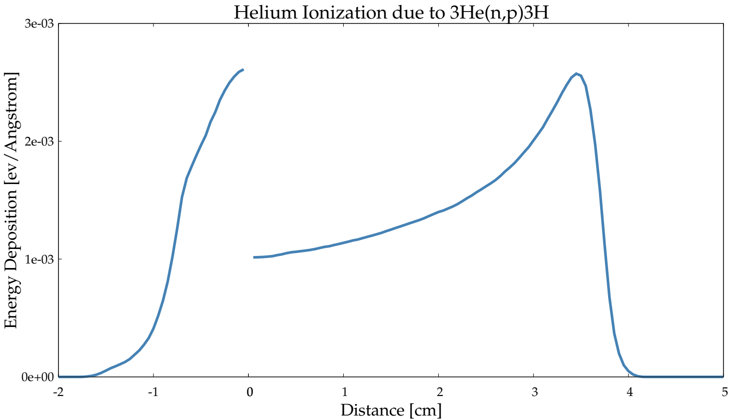
We can now use two one-dimensional generators instead of one two-dimensional one with very good accuracy.

Cross Section



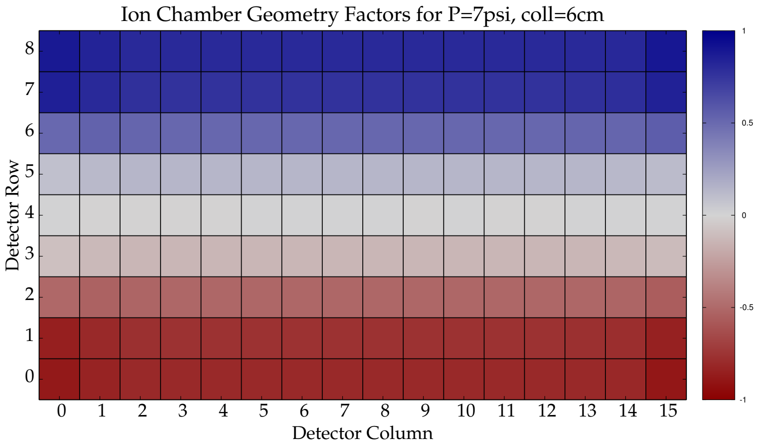
Cross section generated from function, rather than by lookup, by taking advantage of $\frac{1}{v}$ behavior. Linear parameter found by fitting ENDF data to linear function: $C = 2.92709$

Ion Energy Deposition



Energy deposition curve at 1 atm. This is adjusted depending on simulated pressure. Pre-integrate deposition curves and interpolate the difference instead of integrating every time!

Geometry Factors



Optimizations

Calculation of α

There are 144 signal wires in the ion chamber, and each pulse will be divived into 49 time slices. So in total there will be 7056 signal elements for every neutron pulse that reacts in the ion chamber. We will combine each element asymmetry to form the combined physics asymmetry by creating a weighted average:

$$\alpha = \sum_{\kappa} w_{\kappa} \alpha_{\kappa} = \vec{w} \cdot \vec{\alpha}$$

$$\sum_{\kappa} w_{\kappa} = 1$$

We can determine $\vec{w}$ by minimizing the uncertainty in α. So, now we need to define the uncertainty in α: σ_{α}.

Covariance

At a pressure of 0.476 atmosphere, the total ionization range is 12cm. The x-z dimension of one cell is 1.9cm, so the ionization energy of a single reaction can be spread out over several cells. This means we expect to see significant correlation of signals.

Therefore, it is necessary to treat the full covariance matrix in error analysis, rather than calculating only the single-cell uncertainties. So, for any two elements κ and β, define the covariance of their signals to be[2]:

$$\sigma_{\alpha_\kappa \alpha_\beta} = \frac{\langle E^\kappa E^\beta \rangle}{2\langle E^\kappa \cos\theta \rangle \langle E^\beta \cos\theta \rangle}$$

[2] Can be derived through application of Poisson statistics.

Error in α

Apply error propagation to calculate the total error in α from the covariance matrix:

$$\sigma_\alpha^2 = \sum_i \sum_j \frac{\partial \alpha}{\partial \alpha_i} \frac{\partial \alpha}{\partial \alpha_j} \sigma_{\alpha_i \alpha_j} = \sum_i \sum_j w_i w_j \sigma_{\alpha_i \alpha_j} = \vec{w}^{\mathsf{T}} \cdot \hat{\sigma}_{ij} \cdot \vec{w}$$

Here we use the definition of the weight vector. Once we have solved for $\vec{w}$, we can go back and calculate our total uncertainty.

Optimization Function

By applying the constraint to our uncertainty function, we can solve for the weights:

$$\frac{\partial \sigma_\alpha^2}{\partial w_k} = \sum_i w_i \sigma_{\alpha_i \alpha_j} = \lambda_k \frac{\partial(\sum_i w_i - 1)}{\partial w_k} = \mathbf{I}$$

$$\Rightarrow w_i = \sum_j [\sigma_{\alpha_\kappa \alpha_\beta}]_{ij}^{-1}$$

So now we can calculate the total uncertainty in the asymmetry:

$$\boxed{\frac{1}{\sigma_\alpha^2} = \sum_i \sum_j [\sigma_{\alpha_\kappa \alpha_\beta}]_{ij}^{-1}}$$

To optimize the experimental parameters, simulate the uncertainty for different values. We can apply this to the pressure of ^3He and the neutron beam collimation.

Optimization Curves for Pressure and Collimation